Cosmological braneworld solutions with bulk scalar field in DGP setup

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Abstract

We study cosmological dynamics of a canonical bulk scalar field in the DGP setup within a superpotential approach. We show that the normal branch of this DGP-inspired model realizes a late-time de Sitter expansion on the brane. We extend this study to the case that the bulk contains a phantom scalar field. Our detailed study in the supergravity-style analysis reveals some yet unexplored aspects of cosmological dynamics of bulk scalar field in the normal DGP setup. Some clarifying examples along with numerical analysis of the model parameter space are presented in each case.

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1 Introduction

In the revolutionary braneworld viewpoint, our universe is a 3-brane embedded in an extra dimensional bulk. Standard matter and all interactions are confined on the brane; only graviton and possibly non-standard matter are free to probe the full bulk [1,2]. Based on the braneworld viewpoint, our universe may contain many more dimensions than those we experience with our senses. The most compelling reasons to believe in extra dimensions are that they permit new connections between physical properties of the observed universe and suggest the possibility for explaining some of its more mysterious features. Extra dimensions can have novel implications for the world we see, and they can explain phenomena that seem to be mysterious when viewed from the perspective of a three-dimensional observer. Even if one is doubtful about string theory due to, for instance, its huge number of landscapes, recent researches have provided perhaps the most compelling argument in the favor of extra dimensions: a universe with extra dimensions might contain clues to physics puzzles that have no convincing solutions without them. This reason alone makes extra dimensional theories worthy of investigation. In this streamline, the braneworld models that are inspired by ideas from string theory provide a rich and interesting phenomenology, where higher-dimensional gravity effects in the early and late universe can be explored, and predictions can be made in comparison with high-precision cosmological data. Even for the simplest models of RS and DGP, braneworld cosmology brings new implications on the inflation and structure formation [1,3]. Also it brings new ideas for dark energy and opens up exciting prospects for subjecting M-theory ideas to the increasingly stringent tests provided by high-precision astronomical observations [3]. At the same time, braneworld models provide a rich playground for probing the geometry and dynamics of the gravitational field and its interaction with matter [3]. In these respects, the braneworld model of Dvali, Gabadadze and Porrati (DGP) is a scenario that gravity is altered at immense distances by the excruciatingly slow leakage of gravity off our 3-brane universe. In this braneworld scenario, the bulk is considered as empty except for a cosmological constant and the matter fields on the brane are considered as responsible for the evolution on the brane [4,5]. The self-accelerating DGP branch explains late-time speed-up by itself, without recourse to dark energy or other mysterious components [5,6]. Even the normal DGP branch has the potential to realize an effective phantom phase via dynamical screening of the brane cosmological constant [7].

Here we are going to study cosmological dynamics in a DGP setup with a bulk canonical/phantom scalar field. Many authors have studied the cosmological consequences of a bulk scalar field (see for instance [8,9,10]). One of the first motivations to introduce a bulk scalar field was to stabilize the distance between the two branes in the Randall-Sundrum two-brane model [11]. A second motivation for studying scalar fields in the bulk is that such a setup could provide some clue to solve the cosmological constant problem [12]. Models with inflation driven by bulk scalar field have been studied and it is shown that inflation is possible without inflaton on the brane [13]. Generally, the scalar field living in the bulk affects the cosmological dynamics on the brane considerably. The evolution of this field has some interesting cosmological implications; it can give rise self-acceleration and phantom-like phase even in the normal DGP branch of the model in some appropriate situations. Solving

the field equations for a braneworld cosmology with bulk scalar field is not generally an easy task. Nevertheless, during the past decade attempts have been performed to handle this problem. As an attempt, one can express the five-dimensional Einstein equations in terms of 4-dimensional tensors on the brane. Then, by using the Darmois-Israel matching conditions, one can determine these tensors [14]. However, this approach cannot determine all the tensors on the brane. As has been pointed out in Ref. [15], one can proceed further by making assumptions about the bulk solution. But, it is not obvious that such assumptions are justified [9]. To determine which solutions of the 4-dimensional Einstein equations are allowed, one should solve the full 5-dimensional field equations. Full bulk solutions for static braneworlds have been found in some special cases (see [9] and references therein). If the scalar field potential takes a supergravity-like form, the field equations can be reduced to first order equations, which can be solved with relative ease [16] (see also [9]). In this paper, we generalize the work of Davis [9] to the DGP setup. We consider an extension of the DGP scenario that the bulk is non-empty and contains a canonical or phantom scalar field. Since the self-accelerating DGP branch has ghost instabilities, we restrict our study to the normal DGP branch of the model. The bulk equations of motion are derived and some special classes of solutions are presented. We determine the evolution of the brane when the potential of the scalar field takes a supergravity-like form. Some clarifying examples along with numerical analysis of the model parameter space are presented in each case. The importance of this work lies in the fact that bulk scalar field in DGP setup has not been studied in supergravity-style analysis yet. Also our detailed study in this framework reveals some yet unexplored aspects of cosmological dynamics of the bulk scalar field in DGP setup.

2 A canonical bulk scalar field in the DGP setup

2.1 The bulk field equations

The five-dimensional action for a DGP-inspired braneworld model with a bulk canonical scalar field can be written as follows

$$S=\int_{bulk}d^5x\sqrt{-g}\Big\{\frac{1}{2\kappa_5^2}\,{}^{(5)}R-\frac{1}{2}(\nabla\phi)^2-V(\phi)\Big\}-\int_{brane}d^4x\sqrt{-h}\Big(\frac{1}{2\kappa_4^2}R+\frac{1}{2\kappa_5^2}[K]+\mathcal{L}_b(\phi)\Big)\,,\quad(1)$$

where g_{AB} is the bulk metric and h_{AB} is the induced metric on the brane. They are related by $h_{AB}=g_{AB}-n_An_B$, where n_A is the unit vector normal to the 3-brane and A,B are the five dimensional indices. The Gibbons-Hawking boundary term is included via jump of the trace of the extrinsic curvature $[K]$ in the brane action. Also, $\kappa_5^2=\frac{8\pi}{M_5^3}$, where M_5 is the fundamental five-dimensional Planck mass. The brane Lagrangian $\mathcal{L}_b(\phi)$ includes all the Standard Model fields which are confined to the brane, and depends on the bulk scalar field. Varying the action with respect to the bulk scalar field and also the bulk metric gives the

bulk equations of motion

$$\nabla^2\phi = \frac{dV}{d\phi} + \frac{\sqrt{-h}}{\sqrt{-g}}\frac{d\mathcal{L}_b(\phi)}{d\phi}\delta(y)\,, \tag{2}$$

$$G^A_B = \kappa_5^2\left(\nabla^A\phi\nabla_B\phi - \delta^A_B\Big[\frac{1}{2}(\nabla\phi)^2 + V(\phi)\Big]\right) + \delta(y)\kappa_5^2 T^{A(brane)}_B\,, \tag{3}$$

where

$$T^{(brane)}_{AB} = -2\frac{\delta\mathcal{L}_{brane}}{\delta h^{AB}} + h_{AB}\mathcal{L}_{brane}. \tag{4}$$

$T^{(brane)}_{AB}$ is the energy-momentum tensor localized on the brane. $\delta(y)$ is the Dirac delta function with support on the brane which we assume to be located at $y=0$ where y is the coordinate of the extra dimension. The action (1) implies the following jump conditions

$$\Big[N^A\nabla_A\phi\Big] = \frac{\delta\mathcal{L}_b(\phi)}{\delta\phi}\,, \tag{5}$$

$$\Big[K_{AB} - Kh_{AB}\Big] = -\kappa_4^2 T^{(brane)}_{AB}. \tag{6}$$

The Gauss-Codacci equations relate projections of the bulk Einstein tensor to the extrinsic curvature via

$$N^A G_{AB} h^B_C = D_A K^A_C - D_C K\,, \tag{7}$$

where D_A is the covariant derivative with respect to the bulk metric. Combining this with (3) and substituting it into the jump conditions (5) and (6) we find the effective energy-momentum conservation equation on the brane

$$D_A T^{A(brane)}_C = -\frac{\delta\mathcal{L}_b(\phi)}{\delta\phi}D_C\phi. \tag{8}$$

To formulate cosmological dynamics on the brane, we assume the following line element

$$ds^2 = g_{AB}dx^Adx^B = -n^2(y,t)dt^2 + a^2(y,t)\gamma_{ij}dx^idx^j + b^2(y,t)dy^2 \tag{9}$$

where γ_{ij} is a maximally symmetric 3-dimensional metric defined as $\gamma_{ij} = \delta_{ij} + k\frac{x_ix_j}{1-kr^2}$ where $k=-1,0,+1$ parameterizes the spatial curvature and $r^2 = x_ix^i$.

Since we consider here homogeneous and isotropic geometries inside the brane, $T^{A(brane)}_B$ can be expressed quite generally in the following form

$$T^{A(brane)}_B = \frac{1}{b}diag(-\rho_b\,,\,p_b\,,\,p_b\,,\,p_b\,,\,0)\,. \tag{10}$$

The extrinsic curvature tensor in the background metric (9) is given by

$$K^A_B = diag\Big(\frac{n'}{nb}\,,\,\frac{a'}{ab}\delta^i_j\,,\,0\Big). \tag{11}$$

So the jump conditions (5) and (6) are given as follows (we note that in the forthcoming equations, $\frac{\kappa_5^2}{2\kappa_4^2} \equiv r_c$ where r_c is the DGP crossover scale)

$$\frac{[a']}{a_0 b_0} = -\frac{\kappa_5^2}{3}\rho_b + \frac{\kappa_5^2}{\kappa_4^2 n_0^2}\left\{\frac{\dot{a}_0^2}{a_0^2} + k\frac{n_0^2}{a_0^2}\right\} \tag{12}$$

$$\frac{[n']}{n_0 b_0} = \frac{\kappa_5^2}{3}(3p_b + 2\rho_b) + \frac{\kappa_5^2}{\kappa_4^2 n_0^2}\left\{-\frac{\dot{a}_0^2}{a_0^2} - 2\frac{\dot{a}_0\dot{n}_0}{a_0 n_0} + \frac{2\ddot{a}_0}{a_0} - k\frac{n_0^2}{a_0^2}\right\} \tag{13}$$

$$\frac{[\phi']}{b_0} = \frac{\delta\mathcal{L}_b(\phi)}{\delta\phi} \tag{14}$$

where a prime marks differentiation with respect to y and a dot denotes differentiation with respect to t. The subscript 0 marks quantities at $y = 0$ (on the brane). Also $[A] = A(0^+) - A(0^-)$ denotes the jump of the function A across $y = 0$. Assuming Z_2-symmetry about the brane for simplicity, the junction conditions (12)-(14) can be used to compute a', n' and ϕ' on two sides of the brane. The energy-momentum conservation equation (8) on the brane becomes

$$\dot{\rho}_b + 3\frac{\dot{a}_0}{a_0}(\rho_b + p_b) = \frac{\delta\mathcal{L}_b(\phi)}{\delta\phi}\dot{\phi}_0. \tag{15}$$

Because of the presence of the time-dependent bulk scalar field and ϕ-dependent couplings in the standard model lagrangian, the right hand side of the above equation is non-zero and shows the amount of energy non-conservation (due to bulk-brane energy-momentum transfer) of the matter fields on the brane.

2.2 DGP braneworld cosmology with a bulk scalar field

We use the methods presented in Refs. [16,17] (see also [9]) to obtain a special class of solutions for a DGP braneworld cosmology with a bulk scalar field. In this respect, following [17], we introduce the quantity F as a function of t and y as follows

$$F(t,y) = -\Big(\frac{\dot{a}}{an}\Big)^2 + \Big(\frac{a'}{ab}\Big)^2. \tag{16}$$

So, the components of the Einstein tensor can be rewritten in the following simple forms

$$G^0_{\ 0} - \frac{\dot{a}}{a'}G^0_{\ 5} = \frac{3}{2a^3 a'}\partial_y(a^4 F) - \frac{3k}{a^2}, \tag{17}$$

$$G^5_{\ 5} - \frac{a'}{\dot{a}}G^5_{\ 0} = \frac{3}{2a^3 \dot{a}}\partial_t(a^4 F) - \frac{3k}{a^2}. \tag{18}$$

In the presence of the bulk scalar field, the left hand sides of these two equations are not the same. But for special class of solutions with $\phi = \phi(a)$, they are equivalent and in this case $F = F(a)$. In this situation, both (17) and (18) then reduce to

$$\kappa_5^2 V(\phi) + \frac{\kappa_5^2}{2} F\Big(a\frac{d\phi}{da}\Big)^2 + \left\{\frac{3\kappa_5^2}{\kappa_4^2}\Big(\frac{\dot{a}}{a}\Big)^2 + \frac{3\kappa_5^2}{\kappa_4^2}\Big(\frac{k}{a^2}\Big) + \kappa_5^2\,\rho_b\right\}\delta(y) + 6F + \frac{3}{2}a\frac{dF}{da} - \frac{3k}{a^2} = 0. \quad (19)$$

We choose a Gaussian normal coordinate system so that $b^2(y,t) = 1$. Also we assume that t as a proper cosmological time on the brane has scaled so that $n_0 = 1$. By adopting a Z_2 symmetry across the brane, equations (13) and (16) yield the following generalization of the Friedmann equation for cosmological dynamics on the DGP brane

$$\Big(\frac{\dot{a}_0}{a_0}\Big)^2 = \frac{1}{3}\kappa_4^2\rho_b + \frac{2\kappa_4^4}{\kappa_5^4} - \frac{k}{a^2} - \frac{2\kappa_4^2}{\kappa_5^2}\sqrt{\frac{\kappa_4^4}{\kappa_5^4} + \frac{1}{3}\kappa_4^2\rho_b - \frac{k}{a^2} + F_0}. \quad (20)$$

We note that here we consider only the normal, ghost-free branch of the solutions. Since we consider $\phi = \phi(a)$, the field equation (2) reduces to

$$\nabla^2\phi = a^2 F\Big(\frac{d^2\phi}{da^2} + \frac{1}{a}\frac{d\phi}{da}\Big) + \Big[(G^0_{\ 0} + G^5_{\ 5})\frac{a}{3} + \frac{2k}{a}\Big]\frac{d\phi}{da} = \frac{dV}{d\phi} + \delta(y)\frac{\sqrt{-h}}{\sqrt{-g}}\frac{\delta\mathcal{L}_b(\phi)}{\delta\phi}. \quad (21)$$

Substituting equations $G_{AB} = \kappa^2 T_{AB}$ and (19) into (21), we find

$$F\Big(a\frac{d}{da}\Big)^2\phi + \left\{\frac{2a}{3}\Big[\frac{\kappa_5^2}{2}F\Big(a\frac{d\phi}{da}\Big)^2 + \Big\{\frac{3\kappa_5^2}{\kappa_4^2}\Big(\frac{\dot{a}}{a}\Big)^2 + \frac{3\kappa_5^2}{\kappa_4^2}\Big(\frac{k}{a^2}\Big) + \kappa_5^2\,\rho_b\Big\}\delta(y) + 6F + \frac{3}{2}a\frac{dF}{da}\Big] - \frac{k}{a}\right\}\frac{d\phi}{da}$$

$$-\frac{dV}{d\phi} - \delta(y)\frac{\sqrt{-h}}{\sqrt{-g}}\frac{\delta\mathcal{L}_b(\phi)}{\delta\phi} = 0. \quad (22)$$

Thus the original partial differential field equations have been reduced to an ordinary differential equation.

2.3 Supergravity-style solutions

In order to generate some solutions of the field equations, we introduce a special supergravity-style potential, $V(\phi)$, as follows [9]

$$V(\phi) = \frac{1}{8}\Big(\frac{dW}{d\phi}\Big)^2 - \frac{\kappa_5^2}{6}W^2. \quad (23)$$

Assuming $k = 0$, the field equations (19) and (22) are satisfied if

$$F = \frac{\kappa_5^4}{36}W^2, \quad (24)$$

$$a\frac{d\phi}{da} = -\frac{3}{\kappa_5^2 W}\frac{dW}{d\phi}. \quad (25)$$

Here, W is referred to as a superpotential, but we note that supergravity is not required for solutions (23)-(25) to satisfy the equations (19) and (22). Now we can rewrite the Friedmann equation (20) in terms of W as follows

$$\frac{\dot{a}_0^2}{a_0^2}=\frac{1}{3}\kappa_4^2\rho_b+\frac{2\kappa_4^4}{\kappa_5^4}-\frac{2\kappa_4^2}{\kappa_5^2}\sqrt{\frac{\kappa_4^4}{\kappa_5^4}+\frac{1}{3}\kappa_4^2\rho_b+\frac{\kappa_5^4}{36}W_0^2}. \tag{26}$$

From equations (25) and (26) we find the the time variation of the scalar field on the brane as

$$\dot{\phi}^2=\left[\frac{1}{3}\kappa_4^2\rho_b+\frac{2\kappa_4^4}{\kappa_5^4}-\frac{2\kappa_4^2}{\kappa_5^2}\sqrt{\frac{\kappa_4^4}{\kappa_5^4}+\frac{1}{3}\kappa_4^2\rho_b+\frac{\kappa_5^4}{36}W_0^2}\right]\left(\frac{9}{\kappa_5^4W_0^2}\right)\left(\frac{dW}{d\phi}\right)_0^2. \tag{27}$$

The jump conditions (12) and (14) are related via equation (25). Consistency of jump conditions for a Z_2-symmetric DGP brane is guaranteed if

$$\frac{\delta\mathcal{L}_b(\phi)}{\delta\phi}=\left(-\frac{\kappa_5^2}{6}\rho_b+\frac{\kappa_5^2}{2\kappa_4^2}\frac{\dot{a}_0^2}{a_0^2}\right)\left(-\frac{6}{\kappa_5^2W}\frac{dW}{d\phi}\right)_0. \tag{28}$$

So, the energy on the DGP brane is not conserved in this setup. Combining equations (15) and (28), one finds the energy conservation equation in terms of W as follows

$$\dot{\rho}_b+3\frac{\dot{a}_0}{a_0}(\rho_b+p_b)=\left[\rho_b-\frac{3}{\kappa_4^2}\frac{\dot{a}_0^2}{a_0^2}\right]\frac{\dot{W}_0}{W_0} \tag{29}$$

We note that this equation in the case of RSII braneworld scenario has a simpler structure (see for instance, equation (4.7) of Ref. [9]). In our case, due to the presence of two terms on the right hand side of equation (29), there will be new possibilities with different cosmological implications. To discuss the status of the conservation equation in different cases, we define the parameter X so that $X\equiv\frac{3}{\kappa_4^2}\frac{\dot{a}_0^2}{a_0^2}$ for simplicity.

There are three possibilities as follows:

A: $\rho_b > X$

For negative values of $\frac{\dot{W}_0}{W_0}$, energy will leak off the brane. For positive values of this quantity, energy will flow from the bulk into the brane.

B: $\rho_b < X$

In this case for positive values of $\frac{\dot{W}_0}{W_0}$, energy will leak off the brane and for negative values, energy will transfer from the bulk into the brane.

C: $\rho_b = X$

In this case the right hand side of equation (29) vanishes and the sign of $\frac{\dot{W}_0}{W_0}$ is not important, thus the energy on the brane is conserved.

We note that for all mentioned cases, if $\frac{\dot{W}_0}{W_0} = 0$, we have conservation of energy on the brane. Now we consider the simplest generalization of the brane energy density as

$$\rho_b = W_0 \rho \tag{30}$$

where ρ is proportional to the energy density of the ordinary matter on the brane. This generalization has its origin in the fact that matter Lagrangian on the brane, that is $\mathcal{L}_b(\phi)$, depends on the bulk scalar field, ϕ. The effective Friedmann equation (26) then becomes

$$\frac{\dot{a}_0^2}{a_0^2} = \frac{1}{3}\kappa_4^2 W_0\, \rho + \frac{2\kappa_4^4}{\kappa_5^4} - \frac{2\kappa_4^2}{\kappa_5^2}\sqrt{\frac{\kappa_4^4}{\kappa_5^4} + \frac{1}{3}\kappa_4^2 W_0\, \rho + \frac{\kappa_5^4}{36}W_0^2}. \tag{31}$$

This equation implies that if W_0 tends to ∞ at late time, the cosmological evolution on the brane is not generally compatible with observations. On the other hand, for $W_0 = 0$, the result is not compatible with observation too. Only for a constant W_0 we find a viable DGP-like cosmology in this simple generalization. In this case one obtains a self-accelerating solution which explains the late-time cosmic speed up. We note that a phantom-like prescription can be realized in this case in the same way as has been shown in Ref. [18] a canonical scalar field on the brane.

In the next section, we will consider some specific examples of superpotential and we will discuss their cosmological implications. Specifically, we will consider the evolution of W in order to study the status of the conservation equation.

2.4 Some specific examples

We consider the following exponential form of the superpotential [9]

$$W = c\left[\frac{e^{-\alpha_1\phi}}{\alpha_1} + s\frac{e^{\alpha_2\phi}}{\alpha_2}\right], \tag{32}$$

where $s = \pm 1$ and $\alpha_1 \geq |\alpha_2|$. For $\alpha_2 = 0$, we use the following form of the superpotential

$$W = c\left[\frac{e^{-\alpha_1\phi}}{\alpha_1} + \frac{s}{\kappa_5}\right]. \tag{33}$$

The corresponding potentials obtained from (23) are

$$V = \frac{c^2}{8}\left[\left(1 - \frac{4{\kappa_5}^2}{3\alpha_1^2}\right)e^{-2\alpha_1\phi} + \left(1 - \frac{4{\kappa_5}^2}{3\alpha_2^2}\right)e^{2\alpha_2\phi} - 2s\left(1 + \frac{4{\kappa_5}^2}{3\alpha_1\alpha_2}\right)e^{(\alpha_2-\alpha_1)\phi}\right]. \tag{34}$$

and (for $\alpha_2 = 0$)

$$V = \frac{c^2}{8}\left[\left(1 - \frac{4{\kappa_5}^2}{3\alpha_1^2}\right)e^{-2\alpha_1\phi} - 2s\frac{4\kappa_5}{3\alpha_1}e^{-\alpha_1\phi} - \frac{4}{3}\right]. \tag{35}$$

For V bounded from below, only some values of the parameters are allowed [9]. For W as given by (32), one can solve equation (25) to find

$$\ln(\frac{a}{a_*}) = -\frac{{\kappa_5}^2}{3\alpha_1\alpha_2}\ln\left|e^{\alpha_1\phi} - se^{-\alpha_2\phi}\right|, \tag{36}$$

where a_* is an arbitrary constant. For $\alpha_2 = 0$, equation (25) is solved by

$$\ln(\frac{a}{a_*}) = \frac{\kappa_5}{3\alpha_1}(\kappa_5\phi + se^{\alpha_1\phi}). \tag{37}$$

We emphasize here that although the results of this section (equations (36) and (37)) seems to be formally the same as the results obtained in [9] for Randall-Sundrumm II (RSII) braneworld, but one should remember that our Friedmann equation (26) and hence the Hubble parameter and scale factor differ from corresponding quantities in Ref. [9]. With these new quantities, equations (36) and (37) differs essentially from corresponding equations (5.5) and (5.6) of Ref. [9] for RSII case.

Depending on the choice of s and the sign of α_2, there will be a variety of cosmological evolution on the brane with several interesting implications. To illustrate further, in which follows, we will separate each of these subcases for some values of α_1 and α_2. Then we will consider the evolution of scalar field versus the scale factor in normal DGP branch of the scenario.

2.4.1 $\alpha_2 > 0$, $s = +1$

In this case, as a goes from 0 to ∞, scalar field rolls down from either $+\infty$ or $-\infty$ to 0. In figure 1, we show the evolution of the scalar field versus the scale factor explicitly. We fix α_1 in a constant value and choose three different values for α_2. As we see, when the value of α_2 grows, the slope of the curve increases. This increase means that for larger α_2, the scalar field varies faster. Also, for negative ϕ, variation of α_2 has no significant effect on the slope of the curves since all curves coincide in this case. Now we look at the status of the continuity equation in this case.

♣ If we consider the case that ϕ starts from $-\infty$, then as ϕ goes to 0 (and a increases), superpotential evolves from infinity to a constant value with $\dot{W} = 0$ (see figure 5). During this evolution, $\frac{\dot{W}}{W} < 0$, and therefore from (29) we will have the following conditions:

***A**:* If $\rho_b > X$, the right hand side of equation (29) becomes negative and this indicates that energy leaks off the brane as scale factor increases and the universe expands. This situation continues until ϕ and $\dot{W}$ tend to zero. Then, there will be no energy leakage off the brane.

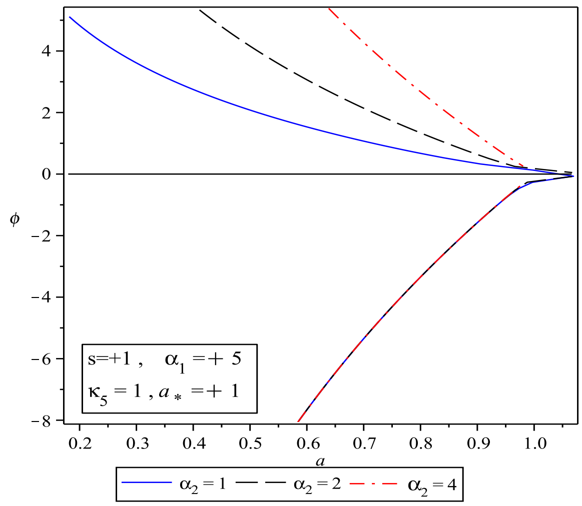

Figure 1: Evolution of the scalar field with respect to the scale factor for the case $\alpha_2 > 0$, $s = +1$. As the scale factor evolves from 0 to ∞, scalar field rolls down from either $+\infty$ or $-\infty$ to 0.

***B**:* If $\rho_b < X$, the right hand side of equation (29) becomes positive and the energy is sucked onto the brane as universe expands. This situation continues until constant value of W ($\dot{W} = 0$) in $\phi = 0$. Then, there will be no energy suction onto the brane.

♣ On the other hand, if we consider the case that ϕ starts from $+\infty$ and tends to zero as a goes to infinity, the superpotential evolves from $+\infty$ to a constant value where $\dot{W} = 0$. During this stage, $\frac{\dot{W}}{W} > 0$. In analogy to the previous case, there are two possibilities:

***A**:* If $\rho_b < X$, the right hand side of equation (29) becomes negative. This means that as the universe expands, energy leaks off the brane until $\dot{W} = 0$.

***B**:* If $\rho_b > X$, the right hand side of the equation of continuity becomes positive and the energy sucks onto the brane as scale factor increases. As soon as $\dot{W}$ vanishes, the energy suction stops.

2.4.2 $\alpha_2 > 0$, $s = -1$

As can be seen from figure 2, for this choice of parameters, ϕ evolves from $+\infty$ to $-\infty$ or vice versa. During this evolution, the scale factor varies from zero to a maximum value and then reduces again to zero. For a constant value of α_1, increasing the value of α_2 leads to increasing in the slope of the curves when ϕ is positive. For negative ϕ, corresponding changes are not significant. Regarding to the continuity equation we have the following possibilities:

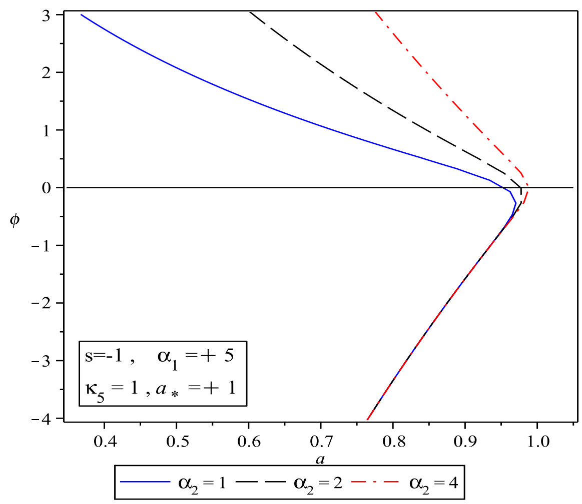

Figure 2: Evolution of the scalar field with respect to the scale factor for the case $\alpha_2 > 0$, $s = -1$. ϕ can evolve from $+\infty$ to $-\infty$ or vice versa. During this evolution, the scale factor changes from zero to a maximum value and then reduces again to zero.

♣ Equation (32) shows that at first, when ϕ is $+\infty$ or $-\infty$, W is $-\infty$ or $+\infty$ respectively (see also figure 5). When ϕ starts to decrease from infinity, we have $\frac{\dot{W}}{W} > 0$. This situation continues until ϕ tends to a special value at which scale factor reaches its maximum value and $\dot{a} = 0$. This implies that:

***A**:* For $\rho_b < X$, energy leaks off the brane as scale factor increases and the universe expands. This leakage of energy continues until the scale factor reaches its maximum value and $\dot{a} = 0$ (see figure 2). At this point, W vanishes and there is nothing left on the brane.

***B**:* If $\rho_b > X$, the situation is very different. When ϕ is $+\infty$ and $\frac{\dot{W}}{W} > 0$, the right hand side of equation (29) is positive. This means that energy is sucked onto the brane whereas the universe expands. This suction persists until scale factor reaches its maximum value.

♣ After scale factor reaches its maximum value, the universe begins to re-collapse and scale factor decreases then. In this stage as ϕ and W goes to $-\infty$ and $+\infty$ respectively, $\frac{\dot{W}}{W} < 0$. This continues until the scale factor returns to zero where the superpotential tends to infinity.

***A**:* If $\rho_b < X$, the right hand side of equation (29) becomes positive. This indicates that as the scale factor decreases and the universe re-collapses, energy is sucked onto the brane. This suction continues until the scale factor tends to zero where W becomes infinity.

***B**:* If $\rho_b > X$, the energy leaks off the brane until the scale factor returns to zero where

the superpotential tends to infinity.

♣ When ϕ starts to increase from $-\infty$, we have $\frac{\dot{W}}{W} < 0$. This situation continues until ϕ tends to a special value at which scale factor reaches its maximum value and $\dot{a} = 0$. This implies that:

***A**:* For $\rho_b > X$, energy leaks off the brane as scale factor increases and the universe expands. This leakage of energy continues until the scale factor reaches its maximum value and $\dot{a} = 0$. At this point, W vanishes.

***B**:* If $\rho_b < X$, the right hand side of equation (29) is positive. This means that energy is sucked onto the brane whereas the universe expands. This suction persists until scale factor reaches its maximum value.

♣ After scale factor reaches its maximum value, the universe begins to re-collapse and scale factor decreases then. In this stage as ϕ goes to $+\infty$, $\frac{\dot{W}}{W} > 0$. This continues until the scale factor returns to zero where the scalar field tends to infinity. So:

***A**:* If $\rho_b > X$, the right hand side of equation (29) becomes positive. This implies that as the scale factor decreases and the universe re-collapses, energy is sucked onto the brane. This suction continues until scale factor tends to zero where ϕ becomes infinity.

***B**:* If $\rho_b < X$, the energy leaks off the brane until the scale factor returns to zero where the scalar field tends to infinity.

2.4.3 $\alpha_2 < 0$, $s = -1$

In this case, the scalar field evolves from $-\infty$ to $+\infty$, as the scale factor goes from zero to ∞. Figure 3 shows this behavior. One can see from this figure that by decreasing the value of α_2, the slope of curves increases for positive scalar field. However, for negative scalar field this change is not significant and there is no considerable shift in curves. Regarding to the continuity equation, the following issues are in order:

♣ At first, when the scalar field is $-\infty$, the superpotential is $+\infty$. Then, by increasing the values of scale factor and scalar field, the superpotential decreases to zero as $\phi \to \infty$. During this evolution, we have $\frac{\dot{W}}{W} < 0$.

***A**:* If $\rho_b > X$, as scale factor increases and the univers expands, energy leaks off the brane and this leakage continues until W tends to zero at late time.

***B**:* If $\rho_b < X$, from equation (29) we find that as the universe expands, the energy is sucked onto the brane.

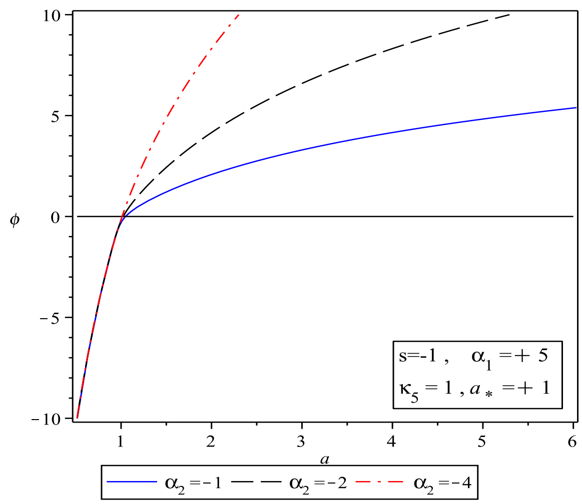

Figure 3: Evolution of the scalar field with respect to the scale factor for the case with $\alpha_2 < 0$ and $s = -1$. As scale factor goes from zero to ∞, the scalar field evolves from $-\infty$ to $+\infty$.

2.4.4 $\alpha_2 < 0$, $s = +1$

For this choice of parameters, the solution starts from $\phi = -\infty$ and $a = 0$. The scalar field increases with scale factor until some value of a where $\dot{a} = 0$. At this point a reaches a relative maximum. After that, the scale factor begins to decrease and scalar field continues its growing until it tends to zero. At this point scale factor reaches its relative minimum, where $\dot{a}$ is zero. Then, ϕ grows again with scale factor towards infinity. This behavior can be seen in figure 4. Further, this figure shows also that decreasing the value of α_2 leads to increasing in the slope of curves for positive ϕ. For negative ϕ, change in the values that α_2 attains has no significant effect on the evolution of ϕ. Once again, regarding to energy conservation in this case we arrive at the following points:

♣ Firstly, when ϕ is $-\infty$, the superpotential is $+\infty$. As ϕ increases and a goes to its relative maximum, W tends to zero. During this stage, $\frac{\dot{W}}{W} < 0$. So we have:

A*:* If $\rho_b > X$, the right hand side of conservation equation (29) becomes negative and energy leaks off the brane with expansion of the universe. This feature continues until W vanishes at the relative maximum of the scale factor.

B*:* If $\rho_b < X$, the right hand side of equation (29) becomes positive and energy is sucked onto the brane as scale factor increases. This situation continues until the scale factor reaches its relative maximum.

♣ Secondly, when a decreases to its relative minimum and ϕ tends to zero, the superpo-

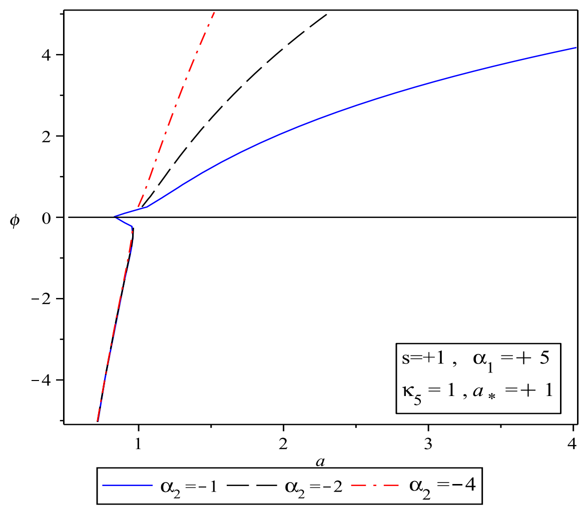

Figure 4: Evolution of the scalar field with respect to the scale factor for the case with $\alpha_2 > 0$ and $s = +1$. The scalar field increases with scale factor until a reaches a relative maximum. After that, the scale factor begins to decrease and the scalar field continues its growing until it tends to zero. Then, ϕ grows to infinity when a tends to infinity.

tential reaches its minimum (here it is a negative value) and $\dot{W}$ vanishes. During this stage, $\frac{\dot{W}}{W} > 0$. We find that:

***A**:* If $\rho_b > X$, the right hand side of equation (29) becomes positive. As the scale factor decreases and the universe re-collapses, energy is sucked onto the brane until $\phi = 0$.

***B**:* If $\rho_b < X$, the right hand side of conservation equation becomes negative. So, the energy leaks off the brane and this leakage persists until the minimum value of superpotential is achieved.

♣ Thirdly, when a starts to increase from its relative minimum towards infinity, the superpotential changes from its minimum to zero when ϕ goes to $+\infty$. During this evolution, we have $\frac{\dot{W}}{W} < 0$. Here also we have:

***A**:* If $\rho_b > X$, the right hand side of equation (29) becomes negative and as a and ϕ tend to infinity, the energy leaks off the brane.

***B**:* If $\rho_b < X$, the right hand side of equation (29) becomes positive. Here, as universe expands, the energy is sucked onto the brane.

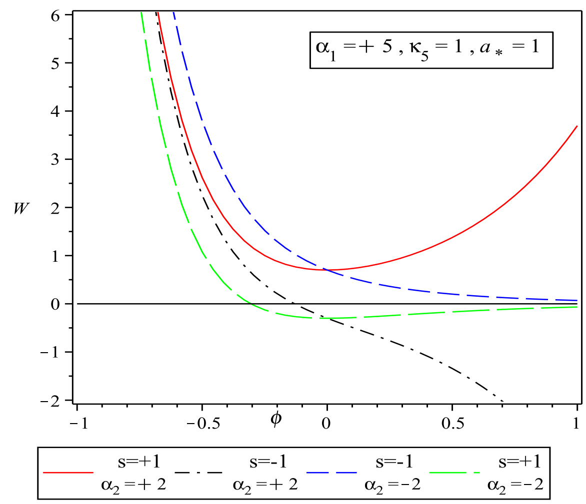

Figure 5: Evolution of the superpotential versus the scalar field. This evolution plays important role in the status of the energy conservation.

2.4.5 $\alpha_2 = 0$, $s = \pm 1$

In these cases, for $s = +1$, scalar field evolves from $-\infty$ to $+\infty$, while for $s = -1$ it evolves from $-\infty$ to $+\infty$ or vice versa. The value of s determines status of this evolution.

♣ If $s = +1$, ϕ starts from $-\infty$ at $a = 0$ and continues to $+\infty$ as a goes to infinity. When ϕ is $-\infty$, the superpotential is $+\infty$. As scalar field increases, the superpotential decreases to a constant value when ϕ tends to infinity. During this stage $\frac{\dot{W}}{W} < 0$. So we have:

A*:* If $\rho_b > X$, as a and ϕ go to infinity, the energy leaks off the brane. This leakage stops when W reaches a constant value.

B*:* If $\rho_b < X$, the right hand side of equation (29) becomes positive. Here, as universe expands, the energy is sucked onto the brane.

♣ If $s = -1$, ϕ starts from $-\infty$ at $a = 0$ and increases until a reaches its maximum. At this point $\dot{a}$ and W tend to zero. In this situation $\frac{\dot{W}}{W} < 0$ and we have:

A*:* If $\rho_b > X$, as a and ϕ go to infinity the energy leaks off the brane. This leakage continues until W tends to zero.

B*:* If $\rho_b < X$, as universe expands, the energy is sucked onto the brane.

Just after $\dot{a}$ becomes zero, the scale factor starts to decrease and the universe re-collapses.

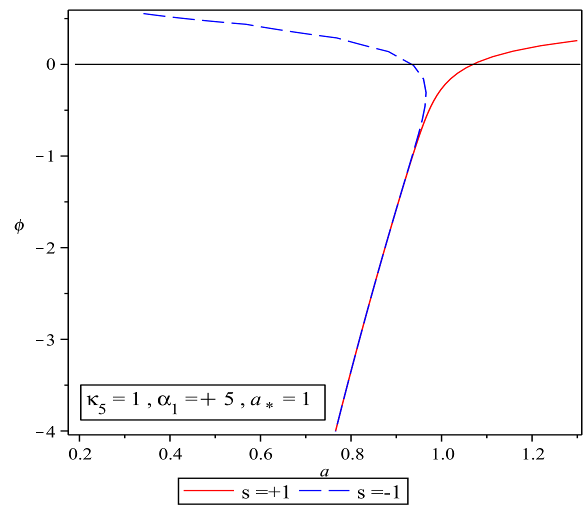

Figure 6: Evolution of scalar field respect to scale factor. If $s = -1$, scale factor has a maximum. But for $s = +1$ it continues to infinity.

When a goes from maximum to zero, the scalar field continues its growth and tends to $+\infty$. During this stage, W decreases and we have $\frac{\dot{W}}{W} > 0$. So the following subcases are in order:

A*:* If $\rho_b < X$, as the universe re-collapses, the energy is leaks off the brane until W tends to a constant value.

B*:* If $\rho_b > X$ as a goes to zero, the energy is sucked onto the brane. This suction persists until the scalar field reaches $+\infty$.

♣ For $s = -1$, there is another situation that ϕ starts from $+\infty$ at $a = 0$ and decrease until a reaches its maximum. At this point $\dot{a}$ and W tend to zero. In this situation $\frac{\dot{W}}{W} > 0$ and we have:

A*:* If $\rho_b < X$, as a increase and ϕ decrease, the energy leaks off the brane. This leakage continues until a reaches its maximum value.

B*:* If $\rho_b > X$, as universe expands, the energy is sucked onto the brane.

Just after $\dot{a}$ becomes zero, the scale factor starts to decrease and the universe re-collapses. When a goes from maximum to zero, the scalar field continues its decrease and tends to $-\infty$. During this stage, W increases and we have $\frac{\dot{W}}{W} < 0$. So the following subcases are in order:

A*:* If $\rho_b > X$, as the universe re-collapses, the energy is leaks off the brane until W tends to a constant value.

B: If $\rho_b < X$ as a goes to zero, the energy is sucked onto the brane. This suction persists until the scalar field reaches $+\infty$.

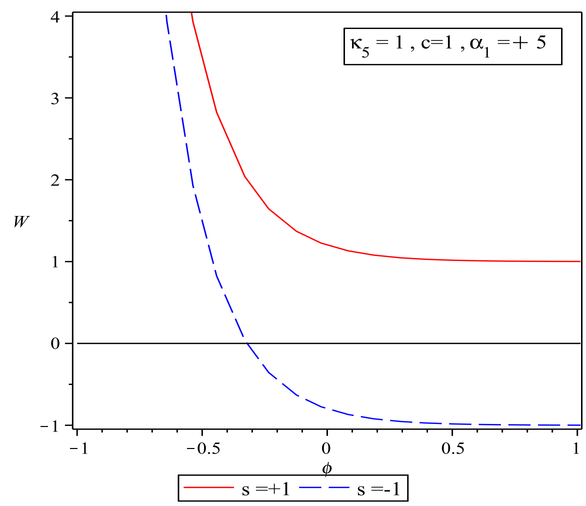

Figure 7: Evolution of the superpotential versus the scalar field for $\alpha_2 = 0$. If $s = -1$, the superpotential will pass through a zero and then tends to a constant . If $s = +1$, the superpotential has no root.

In summary, assuming a simplest generalization of the brane energy density ($\rho_b = W_0 \rho$), implies that if W_0 tends to $+\infty$ at late time, the cosmological evolution on the brane is not generally compatible with observations. On the other hand, for $W_0 = 0$, the result is not compatible with observation too. Only for a constant W_0 we find a viable DGP-like cosmology in this simple generalization. So, we found in this analysis that the case with $\alpha_2 > 0$ and $s = +1$ gives a viable DGP-like cosmology. We should note also that in this case, the situation that the energy leaks of the brane during universe expansion is more reliable.

After a detailed discussion on the energy conservation and possible cosmological dynamics on the brane, in the next subsection we determine cosmological dynamics in the bulk.

2.5 Bulk solutions

By solving the non-zero off-diagonal components of the Einstein field equations, we are able to determine the y-dependence of the metric or the scalar field ϕ in the bulk. To do this end, we assume the fifth dimension is static. In other words, $\dot{b} = 0$ and so we can adopt the gauge $b = 1$. When $\phi = \phi(a)$, we have

$$G_{05} = 3\left(\frac{n'\dot{a}}{na} + \frac{\dot{b}a'}{ba} - \frac{\dot{a}'}{a}\right) = \kappa_5^2 T_{05} = \kappa_5^2 a'\dot{a}\left(\frac{d\phi}{da}\right)^2. \tag{38}$$

Then n can be expressed in terms of a according to the following relation

$$\frac{\dot{a}}{n} = \beta(t)\frac{\kappa_5^2}{6}\exp\left[-\frac{{\kappa_5}^2}{3}\int a\left(\frac{d\phi}{da}\right)^2 da\right] \tag{39}$$

where β is a function of time alone and has no dependence on y. If V has a supergravity-like form as (23), we can rewrite the above expression as

$$\frac{\dot{a}}{n} = \beta(t)\frac{{\kappa_5}^2}{6}W(\phi). \tag{40}$$

We set $n_0 = 1$, then (26) implies that

$$\beta = \frac{a_0}{W_0}\left[\frac{72\kappa_4^4}{\kappa_5^8} + \frac{12\rho_b\kappa_4^2}{\kappa_5^4} - \frac{72\kappa_4^2}{\kappa_5^6}\sqrt{\frac{\kappa_4^4}{\kappa_5^4} + \frac{1}{3}\kappa_4^2\rho_b + \frac{\kappa_5^4}{36}W_0^2}\right]^{1/2}. \tag{41}$$

To have a simple comparison, we note that in the RSII case this quantity is given by $\beta^{(RSII)} = a_0\sqrt{\rho_b^2/W_0^2 - 1}$ (see [9] for instance). Inserting equation (40) into (16) leads to the following differential equation

$$(a')^2 = \frac{\kappa_5^4}{36}(\beta^2 + a^2)W^2(\phi). \tag{42}$$

We take $\alpha_1 = \alpha_2 = \frac{\kappa_5}{\sqrt{3}}$, so that the superpotential used in subsection 2.4 can be simplified to

$$W = \frac{2c}{\alpha_1}\cosh(\alpha_1\phi) = \frac{c\sqrt{3}}{\kappa_5}\left[\left(\frac{a_*}{a}\right)^2 + 4\right]^{\frac{1}{2}}. \tag{43}$$

Therefore the differential equation has a general solution of the form (see [17] for instance)

$$a^2 = A\cosh\mu y + B\sinh\mu y + C, \tag{44}$$

where

$$\mu = \frac{2c\kappa_5}{\sqrt{3}} \tag{45}$$

and the coefficients A, B and C are functions of time.

The Z_2 symmetry across the brane imposes the relations $A_+ = A_- = \bar{A}$ and $B_+ = B_- = \bar{B}$ between these coefficients on the two sides of the brane [17]. From equations (12) and (13) we can find the following relations between coefficients

$$\frac{\bar{B}\mu}{\bar{A} + C} = -\frac{\kappa_5^2}{3}\rho_b + \frac{\kappa_5^2}{\kappa_4^2}\frac{\dot{a}_0^2}{a_0^2} \tag{46}$$

and

$$2\frac{\dot{\bar{B}}\mu}{\dot{\bar{A}} + \dot{C}} - \Big(\frac{\beta\kappa_5^2 W_0}{3}\Big)^2\Big(\frac{\bar{B}\mu}{(\dot{\bar{A}} + \dot{C})^2}\Big) = \frac{\kappa_5^2}{3}(3p_b + 2\rho_b) + \frac{\kappa_5^2}{\kappa_4^2}\Big[-\frac{\dot{a}_0^2}{a_0^2} - 2\frac{\dot{a}_0\dot{n}_0}{a_0} + 2\frac{\ddot{a}_0}{a_0}\Big]. \tag{47}$$

Using equations (26), (40) and (44) and taking the limit $y = 0$, one can determine the coefficients as follows

$$C = a_0^2\Big\{1 - \frac{2}{\mu^2}\Big[\Big(\frac{\dot{a}_0}{a_0}\Big)^2 - \frac{\kappa_5^4}{18}W_0^2 + \frac{\kappa_5^2}{\kappa_4^2}\Big(\frac{\dot{a}_0}{a_0}\Big)^2 + \frac{\kappa_5^2}{3}\rho_b\Big]\Big\}, \tag{48}$$

$$\bar{A} = \frac{2a_0^2}{\mu^2}\Big[\Big(\frac{\dot{a}_0}{a_0}\Big)^2 - \frac{\kappa_5^4}{18}W_0^2 + \frac{\kappa_5^2}{\kappa_4^2}\Big(\frac{\dot{a}_0}{a_0}\Big)^2 + \frac{\kappa_5^2}{3}\rho_b\Big], \tag{49}$$

$$\bar{B} = \frac{a_0^2}{\mu}\Big[-\frac{\kappa_5^2}{3}\rho_b + \frac{\kappa_5^2}{\kappa_4^2}\Big(\frac{\dot{a}_0}{a_0}\Big)^2\Big]. \tag{50}$$

Substituting these coefficients into the general solution (44), we obtain the following expression for the bulk behavior of the scale factor

$$a^2 = a_0^2\Big[\frac{2}{\mu^2}\Big(\Big(\frac{\dot{a}_0}{a_0}\Big)^2 - \frac{\kappa_5^4}{18}W_0^2 + \frac{\kappa_5^2}{\kappa_4^2}\Big(\frac{\dot{a}_0}{a_0}\Big)^2 + \frac{\kappa_5^2}{3}\rho_b\Big)\Big(\cosh(\mu y) - 1\Big)\Big]$$
$$+a_0^2\Big[1 - \frac{1}{\mu}\Big(-\frac{\kappa_5^2}{3}\rho_b + \frac{\kappa_5^2}{\kappa_4^2}\Big(\frac{\dot{a}_0}{a_0}\Big)^2\Big)\sinh(\mu|y|)\Big]. \tag{51}$$

As we mentioned previously, $W \propto a^{-1}$ and therefore, the singularities in this model occur if $|W| \to \infty$. In figure 8, we plotted the scale factor versus ρ_b and y. This figure shows that for some values of ρ_b and y, there are points in the parameter space that the scale factor vanishes. The position of these singularities varies with cosmic time on the brane. At these points, ϕ', $\dot{\phi}$ and W all are singular and is so T_{Ab}. Thus the singularities at $a^2 = 0$ are naked curvature singularities. Nevertheless, if we have a compact bulk or more other branes at suitable distances from our brane, we can avoid these singularities. Also, if we have the solutions with bounded W, it is possible to elusion from the curvature singularities. Note that in ploting figure 8, we have set $W_0 = 2\rho_b$.

If we take $\alpha_1 = -\alpha_2 = \frac{\kappa_5}{\sqrt{3}}$, the differential equation (42) has a general solution of the form

$$a^2 = Ay^2 + By + C. \tag{52}$$

Following the same procedure as above, one can find the following expression for the bulk scale factor

$$a^2 = a_0^2\Big[1 - \frac{1}{\mu}\Big(1 + \Big(\frac{\dot{a}_0}{a_0}\Big)^2 - \frac{\kappa_5^4}{18}W_0^2 + \frac{\kappa_5^2}{\kappa_4^2}\Big(\frac{\dot{a}_0}{a_0}\Big)^2 + \frac{\kappa_5^2}{3}\rho_b\Big)\Big(-\frac{\kappa_5^2}{3}\rho_b + \frac{\kappa_5^2}{\kappa_4^2}\Big(\frac{\dot{a}_0}{a_0}\Big)^2\Big)y\Big]$$
$$+a_0^2\Big[\Big(\Big(\frac{\dot{a}_0}{a_0}\Big)^2 - \frac{\kappa_5^4}{18}W_0^2 + \frac{\kappa_5^2}{\kappa_4^2}\Big(\frac{\dot{a}_0}{a_0}\Big)^2 + \frac{\kappa_5^2}{3}\rho_b\Big)y^2\Big]. \tag{53}$$

Figure 9 shows variation of a^2 versus ρ_b and y. As this figure shows, the bulk scale factor vanishes in some points. Since T_{AB} is also divergent in these points, there are naked curvature singularities in the bulk.

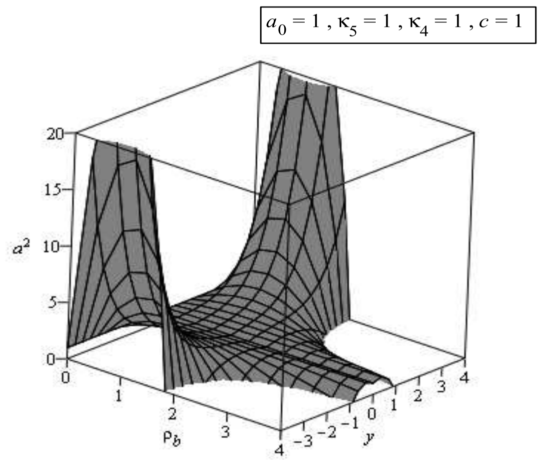

Figure 8: Evolution of the bulk scale factor.

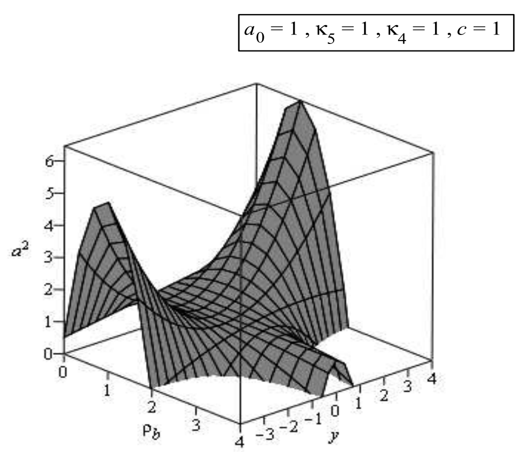

Figure 9: Evolution of the scalar field. For some values of ρ_b and y, it becomes zero. In this points there are naked singularities.

3 Bulk phantom scalar field

3.1 Field equations

In this section we consider a DGP-inspired braneworld model that the bulk contains a phantom scalar field. For a phantom scalar field, the sign of the kinetic energy term is opposite of the canonical scalar field case studied in the previous section. The five-dimensional action for this model can be written as follows

$$S=\int_{bulk}d^5x\sqrt{-g}\Big\{\frac{1}{2\kappa_5^2}R^{(5)}+\frac{1}{2}(\nabla\phi)^2-V(\phi)\Big\}-\int_{brane}d^4x\sqrt{-h}\Big(\frac{1}{2\kappa_4^2}R+\frac{1}{2\kappa_5^2}[K]+\mathcal{L}_b(\phi)\Big)\,. \tag{54}$$

Variation of the above action with respect to the phantom scalar field and also the metric, gives the following equations of motion. These equations differ from equations (2) and (3) for canonical scalar field in the sign of the scalar field dependent terms

$$\nabla^2\phi=-\frac{dV}{d\phi}-\frac{\sqrt{-h}}{\sqrt{-g}}\frac{d\mathcal{L}_b(\phi)}{d\phi}\delta(y)\,, \tag{55}$$

$$G_A^B=\kappa_5^2\bigg(-\nabla^A\phi\nabla_B\phi+\delta_B^A\Big[\frac{1}{2}(\nabla\phi)^2-V(\phi)\Big]\bigg)+\delta(y)\kappa_5^2T_B^{A(brane)}\,. \tag{56}$$

In the presence of the phantom scalar field, the Gauss-Codacci junction conditions are the same as for the canonical scalar field case. Also, if we assume the metric to be as (9), the jump conditions and the energy-momentum conservation equation, are the same as given in section 2.1.

In order to find some solutions of the five dimensional field equations in this case, we use the quantity F and the component of the Einstein tensor used in section 2.2. By using equation (16), for special class of solutions with $\phi=\phi(a)$ and so $F=F(a)$, equations (17) and (18) reduce to

$$\kappa_5^2V(\phi)+\frac{\kappa_5^2}{2}F(a\frac{d\phi}{da})^2+\left\{\frac{3\kappa_5^2}{\kappa_4^2}\Big(\frac{\dot a}{a}\Big)^2+\frac{3\kappa_5^2}{\kappa_4^2}\Big(\frac{k}{a^2}\Big)+\kappa_5^2\,\rho_b\right\}\delta(y)+6F+\frac{3}{2}a\frac{dF}{da}-\frac{3k}{a^2}=0\,. \tag{57}$$

Also the field equation (55) reduces to

$$\nabla^2\phi=a^2F\Big(\frac{d^2\phi}{da^2}+\frac{1}{a}\frac{d\phi}{da}\Big)+\Big[(G^0_{\ 0}+G^5_{\ 5})\frac{a}{3}+\frac{2k}{a}\Big]\frac{d\phi}{da}=-\frac{dV}{d\phi}-\delta(y)\frac{\sqrt{-h}}{\sqrt{-g}}\frac{\delta\mathcal{L}_b(\phi)}{\delta\phi}\,. \tag{58}$$

By using $G_{AB}=\kappa^2T_{AB}$ and equation (58), equation (57) gives

$$F(a\frac{d}{da})^2\phi+\left\{-\frac{2a}{3}\Big[\frac{\kappa_5^2}{2}F(a\frac{d\phi}{da})^2+\Big\{\frac{3\kappa_5^2}{\kappa_4^2}(\frac{\dot a}{a})^2+\frac{3\kappa_5^2}{\kappa_4^2}(\frac{k}{a^2})+\kappa_5^2\,\rho_b\Big\}\delta(y)+6F+\frac{3}{2}a\frac{dF}{da}\Big]+\frac{5k}{a}\right\}\frac{d\phi}{da}$$

$$+\frac{dV}{d\phi}+\delta(y)\frac{\sqrt{-h}}{\sqrt{-g}}\frac{\delta\mathcal{L}_b(\phi)}{\delta\phi}=0. \tag{59}$$

This is the effective field equation for a bulk phantom scalar field in the DGP setup. In which follows, we present some supergravity-style solutions for this effective field equation.

3.2 Supergravity-style solutions

In this subsection, to present some solutions of the field equations, we use a supergravity-style potential introduced earlier in section 3.2. Assuming $k = 0$, the field equations (57) and (59) are satisfied if

$$F = -\frac{\kappa_5^4}{36} W^2, \tag{60}$$

$$a\frac{d\phi}{da} = -\frac{3}{\kappa_5^2 W}\frac{dW}{d\phi}. \tag{61}$$

respectively. The Friedmann equation on the brane now takes the following form

$$\frac{\dot{a}_0^2}{a_0^2} = \frac{1}{3}\kappa_4^2\rho_b + \frac{2\kappa_4^4}{\kappa_5^4} - \frac{2\kappa_4^2}{\kappa_5^2}\sqrt{\frac{\kappa_4^4}{\kappa_5^4} + \frac{1}{3}\kappa_4^2\rho_b - \frac{\kappa_5^4}{36}W_0^2}\ . \tag{62}$$

Also, time variation of the phantom scalar field now is given as follows

$$\dot{\phi}^2 = \left(\frac{1}{3}\kappa_4^2\rho_b + \frac{2\kappa_4^4}{\kappa_5^4} - \frac{2\kappa_4^2}{\kappa_5^2}\sqrt{\frac{\kappa_4^4}{\kappa_5^4} + \frac{1}{3}\kappa_4^2\rho_b - \frac{\kappa_5^4}{36}W_0^2}\right)\left(\frac{9}{\kappa_5^4 W_0^2}\right)\left(\frac{dW_0}{d\phi}\right)^2. \tag{63}$$

We note that the energy conservation equation is the same as given by equation (29).

3.3 A phantom field superpotential

As discussed in Ref. [19], the curvature of the universe grows toward infinity within a finite time in the universe dominated by a phantom fluid. In the case of a phantom scalar field, this Big Rip singularity may be avoided if the potential has a maximum. Here we consider the following form of the superpotential which has the mentioned property

$$W(\phi) = W_0\left(\cosh\left(\frac{\alpha\phi}{m_{pl}}\right)\right)^{-1} \tag{64}$$

This superpotential has a maximum at $\phi = 0$ and tends to zero when the phantom scalar field grows to infinity (see figure 8). The corresponding potential obtained from equation (23) is

$$V(\phi) = \frac{1}{2}W_0^2\left[\frac{1}{4}\left(\frac{\alpha}{m_{pl}}\right)^2\left(\tanh\left(\frac{\alpha\phi}{m_{pl}}\right)\right)^2 - \frac{\kappa_5^2}{3}\right]\left(\cosh\left(\frac{\alpha\phi}{m_{pl}}\right)\right)^{-2}. \tag{65}$$

The solution of equation (61) when W is given as (64) is as follows

$$\ln\left(\frac{a}{a_*}\right) = \frac{\kappa_5^2}{3}\left(\frac{m_{pl}}{\alpha}\right)^2 \ln\left(\sinh\left(\frac{\alpha\phi}{m_{pl}}\right)\right), \tag{66}$$

where a_* is an arbitrary constant. This relation shows that for a bulk phantom scalar field, the result of the superpotential approach for evolution of the scalar field versus the scale factor, is different for even and odd values of α. For odd values of α, scalar field varies from

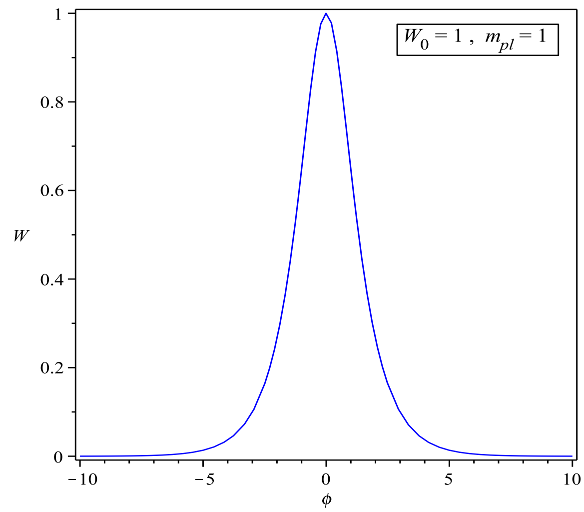

Figure 10: Evolution of the phantomic superpotential (64) with respect to the phantom scalar field. This superpotential has a maximum at $\phi = 0$.

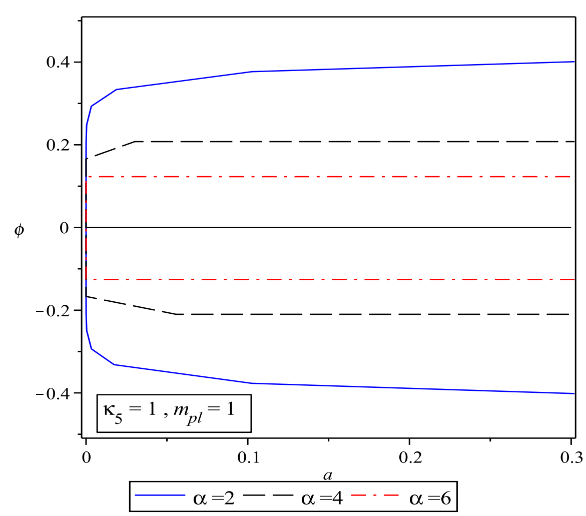

Figure 11: Evolution of the phantom scalar field versus the scale factor for even values of α.

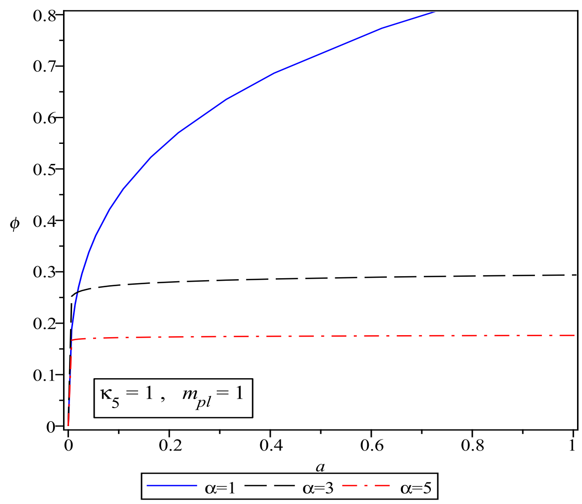

Figure 12: Evolution of the phantom scalar field with respect to the scale factor for odd values of α.

zero to $+\infty$ as scale factor grows to infinity. But, for even values of α, as scale factor goes to infinity, scalar field varies from zero to both $\pm\infty$.

As figure 8 shows, W has its maximum value at $\phi = 0$. In this point, since $\dot{W}_0 = 0$ too, the right hand side of equation (29) vanishes and we have the energy conservation on the brane.

♣ When ϕ goes to $+\infty$, W decreases until it tends to zero when scalar field reaches infinity. During this course, $\frac{\dot{W}}{W} < 0$. This implies that:

A*:* For $\rho_b > X$, energy leaks off the brane as scale factor increases and the universe expands. This leakage of energy continues until the scale factor reaches infinity and W tends to zero. After that there is no leakage of energy-momentum off the brane.

B*:* However, if $\rho_b < X$, the situation is different. Since $\frac{\dot{W}}{W} < 0$, so the right hand side of the conservation equation is positive. This means that energy is sucked onto the brane whereas the universe expands. This suction persists until scale factor and the scalar field reach infinity. After that the suction of energy onto the brane will be stopped.

♣ As ϕ goes from 0 to $-\infty$, W decreases until it tends to zero when scalar field reaches infinity. During this course, $\frac{\dot{W}}{W} > 0$. This implies that:

A*:* For $\rho_b < X$, energy leaks off the brane as scale factor increases and the universe expands. This leakage of energy continues until W tends to zero. After that there is no leakage of energy-momentum off the brane.

B: However, if $\rho_b > X$, the right hand side of the conservation equation is positive. This means that energy is sucked onto the brane whereas the universe expands. This suction persists until scale factor and the scalar field reach infinity. After that the suction of energy onto the brane will be stopped.

We note that since in this case W tends to zero at late time, the model with bulk scalar field with superpotential as given by equation (64), is not a viable DGP-like cosmology.

3.4 Bulk solutions with phantom field

To find the y dependence of the metric (or ϕ), we proceed the method used in section 5.2. As before, we assume the fifth dimension is static ($\dot{b} = 0$). When $\phi = \phi(a)$, we have

$$G_{05} = 3\left(\frac{n'\dot{a}}{na} + \frac{\dot{b}a'}{ba} - \frac{\dot{a}'}{a}\right) = \kappa_5^2 T_{05} = -\kappa_5^2 a'\dot{a}\left(\frac{d\phi}{da}\right)^2. \tag{67}$$

So, we can find the expression of n in terms of a

$$\frac{\dot{a}}{n} = \beta(t)\frac{\kappa_5^2}{6}\exp\left[\frac{\kappa_5^2}{3}\int a\left(\frac{d\phi}{da}\right)^2 da\right]. \tag{68}$$

As before, β has no y-dependence. If V has a supergravity-like form as (23), one can simplify the above equation to

$$\frac{\dot{a}}{n} = -\beta(t)\frac{\kappa_5^2}{6}W(\phi). \tag{69}$$

For $n_0 = 1$, equation (62) gives

$$\beta = -\frac{a_0}{W_0}\left[\frac{12\kappa_4^2\rho_b}{\kappa_5^4} + \frac{72\kappa_4^4}{\kappa_5^8} - \frac{72\kappa_4^2}{\kappa_5^6}\sqrt{\frac{\kappa_4^4}{\kappa_5^4} + \frac{\kappa_4^2\rho_b}{3} - \frac{\kappa_5^4}{36}W_0^2}\right]^{\frac{1}{2}}. \tag{70}$$

By substituting (69) into equation (16), we find

$$(a')^2 = \frac{\kappa_5^4}{36}(\beta^2 - a^2)W^2(\phi). \tag{71}$$

Following the same procedure as adopted in subsection 2.5, and by redefinition of W as equation (64), we find the following expression for the bulk behavior of the scale factor

$$\begin{aligned} a^2 = a_0^2&\left[\frac{2}{\mu^2}\left(\left(\frac{\dot{a}_0}{a_0}\right)^2 - \frac{\kappa_5^4}{18}W_0^2 + \frac{\kappa_5^2}{\kappa_4^2}\left(\frac{\dot{a}_0}{a_0}\right)^2 + \frac{\kappa_5^2}{3}\rho_b\right)\Big(\cosh(\mu y) - 1\Big)\right] \\ &+a_0^2\left[1 - \frac{1}{\mu}\left(-\frac{\kappa_5^2}{3}\rho_b + \frac{\kappa_5^2}{\kappa_4^2}\left(\frac{\dot{a}_0}{a_0}\right)^2\right)\sinh(\mu|y|)\right]. \end{aligned} \tag{72}$$

We note that for a bulk phantom scalar field with a bounded superpotential, as given by (64), the model has no singularity.

4 Summary and Discussion

In this paper, we have studied cosmological dynamics in a DGP setup with a bulk canonical/phantom scalar field. Bulk scalar field is motivated in several context: to stabilize the distance between two branes in the Randall-Sundrum two-brane model, a way to solve the cosmological constant problem, and inflation driven by bulk scalar field without inflaton on the brane. Generally, the scalar field living in the bulk affects the cosmological dynamics on the brane considerably. The evolution of this field has interesting cosmological effects and can give rise self-acceleration and phantom-like phase even in the normal DGP branch of the model in some appropriate situations. In this paper, we have generalized the work of Ref. [9] to the DGP setup. We considered an extension of the DGP scenario that the bulk is non-empty and contains a canonical or phantom scalar field. Since the self-accelerating DGP branch has ghost instabilities, we restricted our study to the normal DGP branch of the model. The bulk equations of motion are derived and some special classes of the solutions are presented. We determined also the evolution of the brane when the potential of the scalar field takes a supergravity-like form. Some clarifying examples along with numerical analysis of the model parameters space are presented in each step. The importance of this work is that bulk scalar field in the DGP setup was not studied in supergravity-style analysis. Also our detailed study in this framework has revealed some yet unexplored aspects of cosmological dynamics of the bulk scalar field in DGP setup. We have extended this study to the case that the bulk contains a phantom scalar field too. Our strategy to perform the mentioned analysis was as follows:

First of all, from a five-dimensional action for a DGP-inspired braneworld model with a bulk canonical scalar field, we found the bulk equations of motion, jump conditions and the Gauss-Codacci equations. Then, using these equations, we achieved the effective energy-momentum conservation equation on the brane. We saw that because of the presence of the time-dependent bulk scalar field and ϕ-dependent couplings in the standard model Lagrangian, the right hand side of the continuity equation is non-zero, showing the amount of energy non-conservation (due to bulk-brane energy-momentum transfer) of the matter fields on the brane. To obtain a special class of solutions for a DGP braneworld cosmology with a bulk scalar field, we used the methods presented in Refs. [9,16,17]. Using that method and introducing the quantity F as a function of t and y, we reduced the original partial differential field equations to an ordinary differential equation. In order to generate some solutions of the field equations, we introduced a special supergravity-style potential $V(\phi)$, including the superpotential W. With this supergravity-style potential, we derived the energy conservation equation on the brane in terms of W. In our model, due to the presence of two terms on the right hand side of the conservation equation (29), there was new possibilities with different cosmological implications. If the right hand side of the conservation equation becomes negative, the energy leaks off the brane. However, if the right hand side of this equation becomes positive, there is energy suction onto the brane. For vanishing right hand side of the conservation equation, energy is conserved on the brane. We considered some specific examples of superpotential and discussed their cosmological implications. Evolution of the scalar filed versus the scale factor, and evolution of W in terms of the scalar field are

discussed fully to study the status of the conservation equation on the brane. Assuming the simplest generalization of the brane energy density ($\rho_b = W_0\rho$), implies that if W_0 tends to $+\infty$ at late time, the cosmological evolution on the brane is not generally compatible with observations. On the other hand, for $W_0 = 0$, the result is not compatible with observation too. Only for a constant W_0 we find a viable DGP-like cosmology in this simple generalization. So, we found in those examples that the case with $\alpha_2 > 0$ and $s = +1$ (subsection 2.4.1) is a viable DGP-like cosmology. Of course in the mentioned case, the situation that the energy leaks of the brane during universe expansion is more favorable. We also determined the bulk behavior of the metric. We saw that since W is not bounded, the naked curvature singularities in the bulk are present. However, with a compact bulk or existence of more other branes at suitable distances from our brane, we can avoid these singularities. Also, for the solutions with bounded W, it is possible to elusion from the curvature singularities. We continued our treatment by considering the bulk phantom scalar field. From the five-dimensional action for a DGP-inspired braneworld model with a bulk phantom scalar field, we found the bulk equations of motion. In the presence of the bulk phantom scalar field, the Gauss-Codacci junction conditions and also the energy-momentum conservation equation, are the same as for the canonical scalar field case. Introducing quantity F in terms of metric components as before, we achieved the effective field equation for a bulk phantom scalar field in the DGP setup. To present some solutions of the field equations, we used a supergravity-style potential introduced earlier in the case of canonical scalar field but with a new superpotential. We considered the evolution of the scalar field versus the scale factor and the evolution of W versus ϕ in order to study the status of the conservation equation on the brane. Since in this case W tends to zero at late time, this case is not a viable DGP-like cosmology. Also, we determined the bulk behavior of the metric. Since in this case W is bounded, there is no singularity in the brane.

Finally the following issues are important to note:

- To be a cosmologically viable scenario, this model with bulk scalar field should explain at least the late-time cosmic speed-up on the brane. As we have shown, this model accounts for cosmic speed-up on the brane in some specific situations. For instance, the case with $\alpha_2 > 0$ and $s = +1$ in subsection 2.4.1 gives a viable DGP-like cosmology explaining late-time cosmic acceleration. Also it is possible to realize an effective phantom-like prescription on the brane without need to phantom matter in the same way as has been done in Ref. [18] for a canonical scalar field on the brane.

- As has been mentioned by Flanagan *et al.* (in Ref. [8]), the principal problem associated with the introduction of a bulk scalar field in models such as the present one is that generic stationary solutions contain timelike curvature singularities in the bulk at finite distances from the branes. One approach to overcome this problem is to simply orbifold or otherwise compactify the fifth dimension in so that the singularity is never encountered. A second approach is to carefully choose the scalar field potential in such a way that the occurrence of singularities is prevented. Here we have

adopted the second strategy by choosing bounded superpotentials. For instance, with a bulk phantom scalar field with a bounded superpotential, the model has no singularity.

- As has been shown in Refs. [12] and [13], in the presence of a bulk scalar field, realization of the inflation is possible without inflaton field on the brane. In fact, inflation can be driven just by a bulk scalar field. In our setup, it is possible to realize inflation in the same way as has been adopted in Refs. [12] and [13]. In fact, the late-time behavior of the bulk scalar field can be treated by analyzing the property of a retarded Green function. Including the lowest order back-reaction to the geometry, this late-time behavior can be well approximated by an effective four dimensional scalar field on the brane. As has been shown by Himemoto *et al.* in Refs. [12,13], the mapping to the four-dimensional effective theory is given by a simple scaling of the potential with a re-definition of the field. This effective four-dimensional field can drive inflation on the brane. This issue is under study and will be addressed in one of our forthcoming report.

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